Dihedral angle is not a normal difference on a manifold mesh

(a small pitfall)

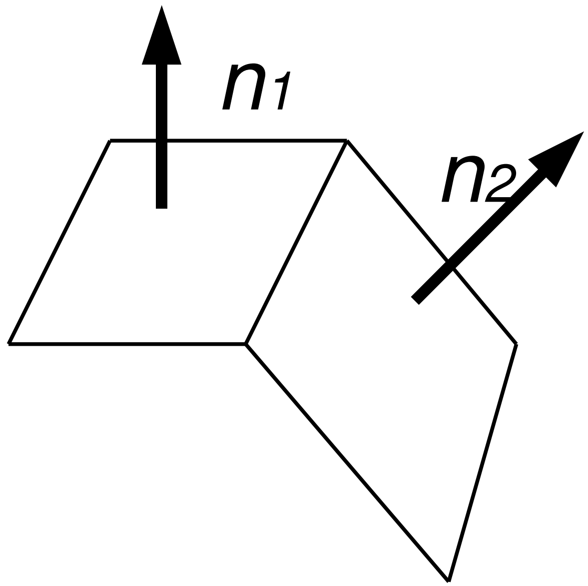
aGF: a Geometry Fan

Berlin, 2007-07-12

Yamauchi, Hitoshi

aGF:

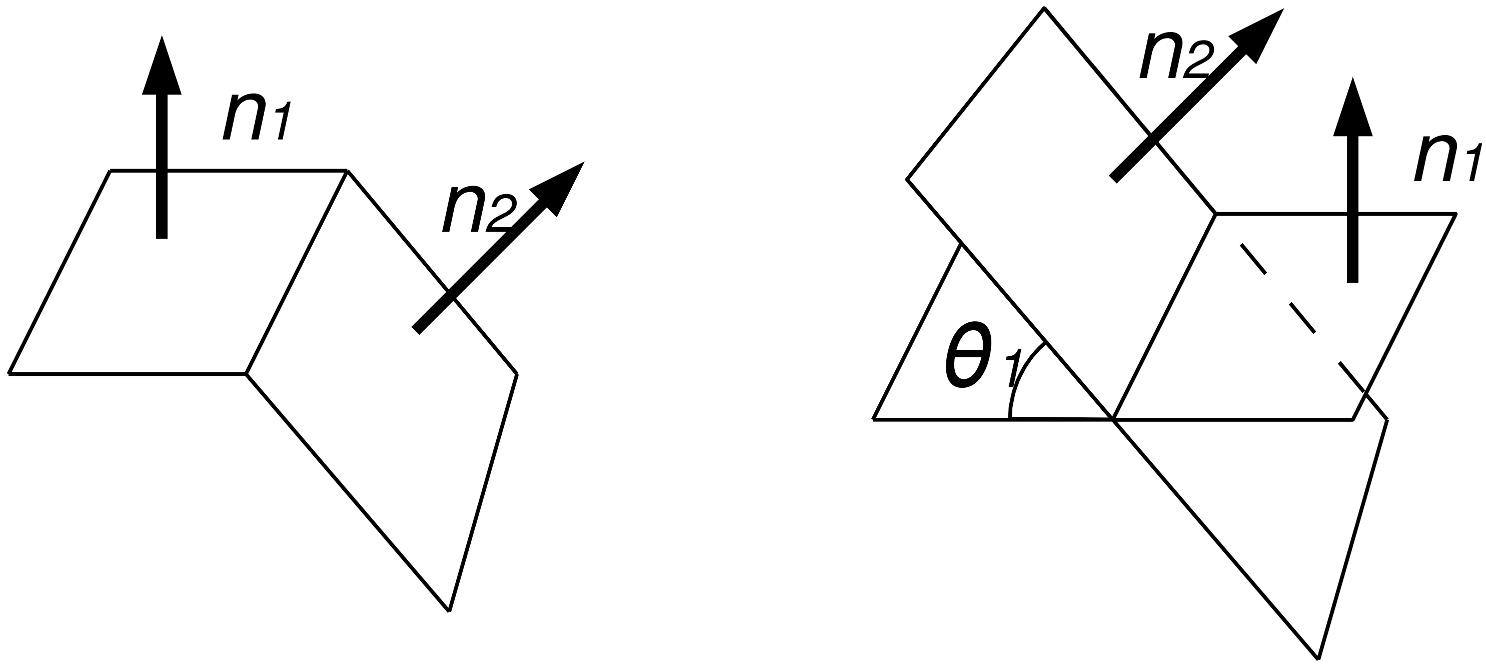
Dihedral angle on a mesh



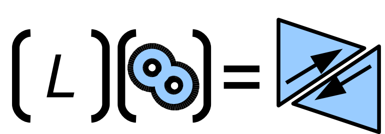
- The dihedral angle is the angle between two planes.
- If you know two normals of plane, $\cos(\theta) = (n_1 \cdot n_2)/(|n_1||n_2|)$

aGF: (L)( ) =

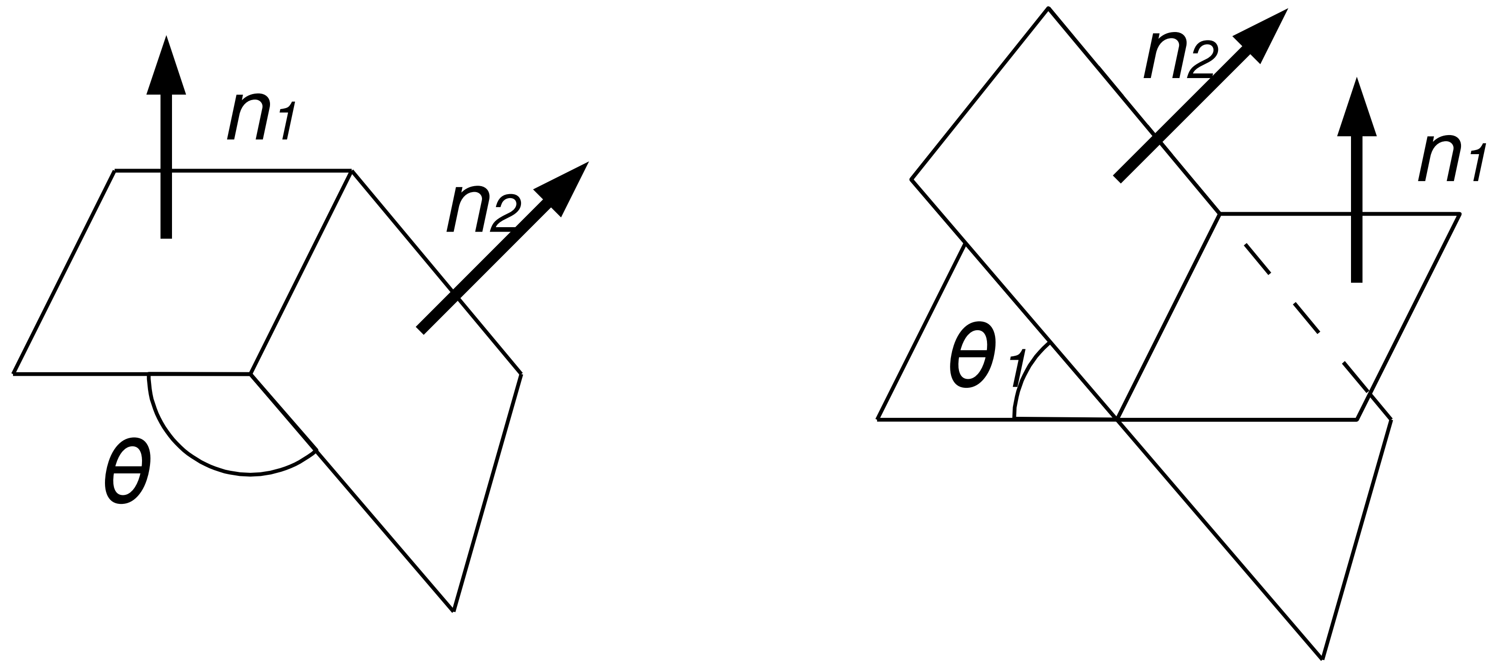
Definition of dihedral angle



- The definition of the dihedral angle is θ_1 [1].

aGF: 

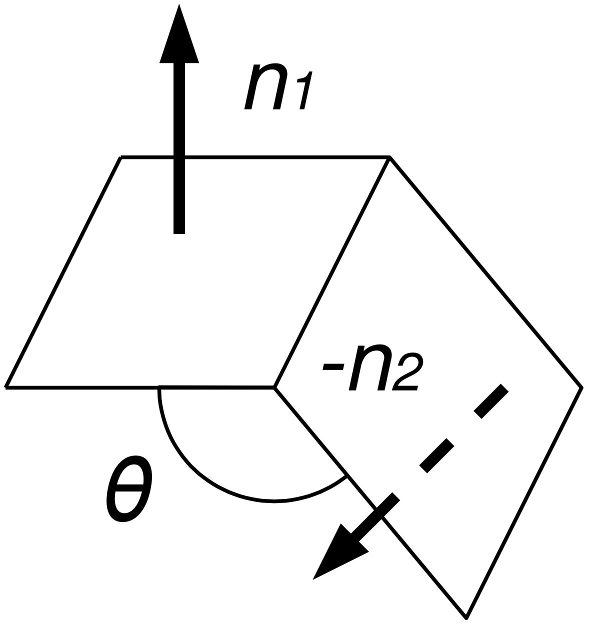
Definition of dihedral angle



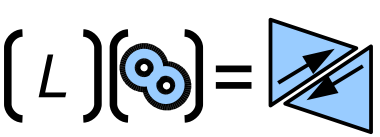
- The definition of the dihedral angle is θ_1 [1].
- If you have a mesh, θ is the dihedral angle.
- They are not the same.

aGF:

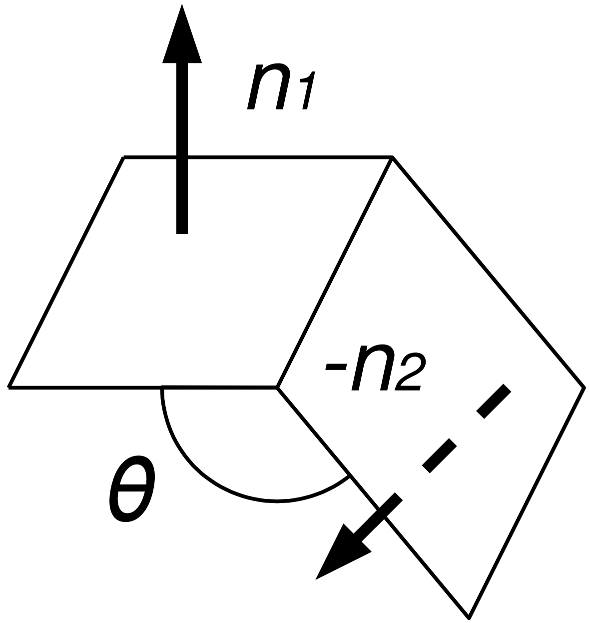
Dihedral angle

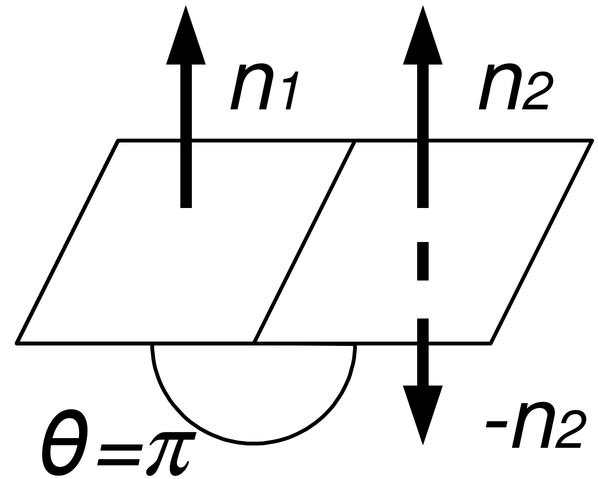


- You need to flip one normal for θ.

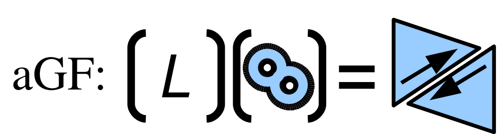


Dihedral angle





- You need to flip one normal for θ.
- If $n_1 = n_2$, there should be no angle, $(n_1 \bullet n_2) = 1$, then $\theta = 0$. (n_1, n_2 are normalized.) But here θ should be π.



A small pitfall about dihedral angle

- Need one normal flip on a manifold mesh.
- Then you can correctly compute the mean curvature over the edge integration [2,3].
- (This memo is motivated by a stupid mistake I made. How I was surprised when I saw a plane mesh has large mean curvature!)

References

- [1] http://mathworld.wolfram.com/
- [2] Polthier K.: Polyhedral surfaces of constant mean curvature. Habilitationsschrift Technische Universität Berlin (2002).
- [3] Cohen-Steiner D., Morvan J.-M.: Restricted Delaunay triangulations and normal cycles. Proc. 19th Annu. ACM Sympos. Comput. Geom. (2003), 237–246.

- (This is not so rigid research report. So, please let me know if there are better references to be added.)

aGF: